Keystone Algebra 1
Information and Practice
2012-13

School District of Haverford Township
Algebra 1 Keystone Exam

The Keystone Exams are end-of-course assessments designed to assess proficiency in the subject areas of Algebra I, Literature, and Biology. This booklet contains sample problems related to the skills and concepts assessed on the Algebra 1 Exam.

Additional resources can be found on Haverford High School’s web page. Select programs and services, then mathematics then Keystone Algebra 1.

### ALGEBRA I KEYSTONE EXAM BREAKDOWN OF QUESTION TYPES

<table>
<thead>
<tr>
<th></th>
<th>Module 1</th>
<th>Module 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Operational Questions</td>
<td>18 3</td>
<td>18 3</td>
<td>36 6</td>
</tr>
<tr>
<td>Number of Field Test Questions</td>
<td>5 1</td>
<td>5 1</td>
<td>10 2</td>
</tr>
<tr>
<td>Total</td>
<td>23 4</td>
<td>23 4</td>
<td>46 8</td>
</tr>
</tbody>
</table>

### ALGEBRA I KEYSTONE EXAM BREAKDOWN OF POINTS

<table>
<thead>
<tr>
<th></th>
<th>Module 1</th>
<th>Module 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Operational Multiple-Choice Questions</td>
<td>18 questions at 1 point each = 18 points</td>
<td>18 questions at 1 point each = 18 points</td>
</tr>
<tr>
<td>Number of Operational Constructed-Response Questions</td>
<td>3 questions at 4 points each = 12 points</td>
<td>3 questions at 4 points each = 12 points</td>
</tr>
<tr>
<td>Total Number of Points</td>
<td>30 points</td>
<td>30 points</td>
</tr>
<tr>
<td>Percentage of Points for Entire Test</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

There will be a total of 60 points (Module 1 and Module 2 combined), with approximately 60% multiple-choice points and 40% constructed-response points.

A student needs to get 34 out of the 60 possible points in order to be proficient on the Algebra 1 Exam.
**ALGEBRA I FORMULA SHEET**

Formulas that you may need to solve questions on this exam are found below. You may use calculator $\pi$ or the number 3.14.

---

**Linear Equations**

- **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$
- **Point-Slope Formula:** $(y - y_1) = m(x - x_1)$
- **Slope-Intercept Formula:** $y = mx + b$
- **Standard Equation of a Line:** $Ax + By = C$

---

**Arithmetic Properties**

- **Additive Inverse:** $a + (-a) = 0$
- **Multiplicative Inverse:** $a \cdot \frac{1}{a} = 1$
- **Commutative Property:**
  - Addition: $a + b = b + a$
  - Multiplication: $a \cdot b = b \cdot a$
- **Associative Property:**
  - Addition: $(a + b) + c = a + (b + c)$
  - Multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- **Identity Property:**
  - Addition: $a + 0 = a$
  - Multiplication: $a \cdot 1 = a$
- **Distributive Property:**
  - $a \cdot (b + c) = a \cdot b + a \cdot c$
- **Multiplicative Property of Zero:** $a \cdot 0 = 0$
- **Additive Property of Equality:**
  - If $a = b$, then $a + c = b + c$
- **Multiplicative Property of Equality:**
  - If $a = b$, then $a \cdot c = b \cdot c$
Module 1

A1.1.1 Operations with Real Numbers and Expressions

- Compare and/or order any real numbers. Note: Rational and irrational may be mixed.
- Simplify square roots.
- Find the Greatest Common Factor (GCF) and/or the Least Common Multiple (LCM) for sets of monomials.
- Simplify/evaluate expressions involving properties/laws of exponents, roots, and/or absolute values to solve problems. Note: Exponents should be integers from -10 to 10.
- Use estimation to solve problems.
- Add, subtract, and/or multiply polynomial expressions (express answers in simplest form). Note: Nothing larger than a binomial multiplied by a trinomial.
- Factor algebraic expressions, including difference of squares and trinomials. Note: Trinomials are limited to the form $ax^2 + bx + c$ where $a$ is equal to 1 after factoring out all monomial factors.
- Simplify/reduce a rational algebraic expression.

Sample Problems:

1. Which of the following inequalities is true for **ALL** real values of $x$?
   a. $x^3 \geq x^2$
   b. $3x^2 \geq 2x^3$
   c. $(2x)^2 \geq 3x^2$
   d. $3(x - 2)^2 \geq 3x^2 - 2$

2. An expression is shown to the right: $2\sqrt{51}x$
   Which value of $x$ makes the expression equivalent to $10\sqrt{51}$
   a. 5
   b. 25
   c. 50
   d. 100

3. An expression is shown below
\[
\sqrt{87x}
\]

For which value of \( x \) should the expression be further simplified?

a. \( x = 10 \)
b. \( x = 13 \)
c. \( x = 21 \)
d. \( x = 38 \)

4. Two monomials are shown below.

\[
450x^2y^5 \quad 3,000x^4y^3
\]

What is the least common multiple (LCM) of these monomials?

a. \( 2xy \)
b. \( 30xy \)
c. \( 150x^2y^3 \)
d. \( 9,000x^4y^5 \)

5. Simplify: \( 2(2\sqrt{4})^{-2} \)

a. \( \frac{1}{8} \)
b. \( \frac{1}{4} \)
c. \( 16 \)
d. \( 32 \)

6. A theme park charges $52 for a day pass and $110 for a week pass. Last month, 4,432 day passes were sold and 979 week passes were sold. Which is the closest estimate of the total amount of money paid for the day and week passes for last month?

a. $300,000
b. $400,000
c. $500,000
d. $600,000

7. A polynomial expression is shown below.
\[(mx^3 + 3)(2x^2 + 5x + 2) - (8x^5 + 20x^4)\]

The expression is simplified to \(8x^3 + 6x^2 + 15x + 6\). What is the value of \(m\)?

a. -8  
b. -4  
c. 4  
d. 8

8. \((x + 3)(2x^2 + 5x + 2)\)

The above expression simplifies to

a. \(2x^3 + 11x^2 + 17x + 6\)  
b. \(2x^2 + 6x + 5\)  
c. \(9x^2 + 21x + 6\)  
d. \(2x^3 + 5x^2 + 6\)

9. When the expression \(x^2 - 3x - 18\) is factored completely, which is one of its factors?

a. \((x-2)\)  
b. \((x-3)\)  
c. \((x-6)\)  
d. \((x-9)\)

10. Simplify: \(\frac{-3x^3 + 9x^2 + 30x}{-3x^3 + 18x^2 - 24x}; \quad x \neq -4, -2, 0\)

a. \(\frac{-1}{2}x^2 - \frac{5}{4}x\)  
b. \(\frac{1}{2}x^2 - \frac{5}{4}x\)  
c. \(\frac{x+5}{x-4}\)  
d. \(\frac{x-5}{x+4}\)
A1.1.2 Linear Equations

- Write, solve, and/or apply a linear equation (including problem situations).
- Use and/or identify an algebraic property to justify any step in an equation-solving process. Note: Linear equations only.
- Interpret solutions to problems in the context of the problem situation. Note: Linear equations only.
- Write and/or solve a system of linear equations (including problem situations) using graphing, substitution, and/or elimination. Note: Limit systems to two linear equations.
- Interpret solutions to problems in the context of the problem situation. Note: Limit systems to two linear equations.

Sample Problems:

11. Jenny has a job that pays her $8 per hour plus tips (t). Jenny worked for 4 hours on Monday and made $65 in all. Which equation could be used to find t, the amount Jenny made in tips?
   a. $65 = 4t + 8$
   b. $65 = 8t ÷ 4$
   c. $65 = 8t + 4$
   d. $65 = 8(4) + t$

12. One of the steps Jamie used to solve an equation shown below.
   \[-5(3x + 7) = 10\]
   \[-15x + -35 = 10\]

   Which statements describe the procedure Jamie used in this step and identify the property that justifies the procedure?
   a. Jamie add -5 and 3x to eliminate the parentheses. This procedure is justified by the associative property.
   b. Jamie added -5 and 3x to eliminate the parentheses. The procedure is justified by the distributive property.
   c. Jamie multiplied 3x and 7 by -5 to eliminate the parentheses. The procedure is justified by the associative property.
   d. Jamie multiplied 3x and 7 by -5 to eliminate the parentheses. The procedure is justified by the distributive property.
13. Francisco purchased $x$ hot dogs and $y$ hamburgers at a baseball game. He spent a total of $10. The equation below describes the relationship between the number of hot dogs and the number of hamburgers purchased.

$$3x + 4y = 10$$

The ordered pair $(2,1)$ is the solution to the equation. What does the solution represent?

a. Hamburgers cost 2 times as much as hot dogs
b. Francisco purchased 2 hot dogs and 1 hamburger
c. Hot dogs cost $2 each and hamburgers cost $1 each
d. Francisco spent $2 on a hot dogs and $1 on a hamburgers

14. Anna burned 15 calories per minute running for $x$ minutes and 10 calories per minute hiking for $y$ minutes. She spent a total of 60 minutes running and hiking and burned 700 calories. The system of equations shown below can be used to determine how much time Anna spent on each exercise.

$$15x + 10y = 700$$
$$x + y = 60$$

What is the value of $x$, in minutes Anna spent running?

a. 10
b. 20
c. 30
d. 40

15. Samantha and Maria purchased flowers. Samantha purchased 5 roses for $x$ dollars each and 4 daisies for $y$ dollars each and spent $32 on the flowers. Maria purchased 1 rose for $x$ dollars and 6 daisies for $y$ dollars each and spent $22. The system of equations below represents this situation.

$$5x + 4y = 32$$
$$x + 6y = 22$$

Which statement is true?

a. A rose costs $1 more than a daisy.
b. Samantha spent $4 on each daisy.
c. Samantha spent more on daisies than she did on roses.
d. Samantha spent over 4 times as much on daisies as she did on roses.
A1.1.2 Linear Inequalities

- Write or solve compound inequalities and/or graph their solution sets on a number line (may include absolute value inequalities).
- Identify or graph the solution set to a linear inequality on a number line.
- Interpret solutions to problems in the context of the problem situation. Note: Limit to linear inequalities.
- Write and/or solve a system of linear inequalities using graphing. Note: Limit systems to two linear inequalities.
- Interpret solutions to problems in the context of the problem situation. Note: Limit systems to two linear inequalities.

Sample Problems:

16. A compound inequality is shown to the right. \( 5 < 2 - 3y < 14 \)
   What is the solution set of the compound inequality?
   a. \(-4 > y > -1\)
   b. \(-4 < y < -1\)
   c. \(1 > y > 4\)
   d. \(1 < y < 4\)

17. The solution set of an inequality is graphed on the number line below.

   ![Number Line with Inequality Solution Set]

   The graph shows the solution set of which inequality?
   a. \(2x + 5 < -1\)
   b. \(2x + 5 \leq -1\)
   c. \(2x + 5 > -1\)
   d. \(2x + 5 \geq -1\)
18. A baseball team had $1,000 to spend on supplies. The team spent $185 on a new bat. New baseballs cost $4 each. The inequality $185 + 4b \leq 1,000$ can be used to determine the number of new baseballs (b) that the team can purchase. Which statement about the number of new baseballs that can be purchased is true?
   a. The team can purchase 204 new baseballs.
   b. The minimum number of new baseballs that can be purchased is 185.
   c. The maximum number of new baseballs that can be purchased is 185.
   d. The team can purchase 185 new baseballs, but this number is neither the maximum nor the minimum.

19. Mike always leaves a tip of between 8% and 20% for the server when he pays for his dinner. This can be represented by the system of inequalities shown below, where y is the amount of tip and x is the cost of the dinner.

   \[
   y > 0.08x \\
   y < 0.2x
   \]

   Which of the following is a true statement?
   a. When the cost of the dinner (x) is $10, the amount of the tip (y) must be between $2 and $8.
   b. When the cost of the dinner (x) is $15, the amount of the tip (y) must be between $1.20 and $3.00.
   c. When the cost of the tip (y) is $3, the amount of the dinner (x) must be between $11 and $23.
   d. When the cost of the tip (y) is $2.40, the amount of the dinner (x) must be between $3 and $6.
20.
A system of inequalities is shown below.

\[ y < x - 6 \]
\[ y > -2x \]

Which graph shows the solution set of the system of inequalities?

A. 

B. 

C. 

D.
Module 2

A1.2.1 Functions

- Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
- Determine whether a relation is a function, given a set of points or a graph.
- Identify the domain or range of a relation (may be presented as ordered pairs, a graph, or a table).
- Create, interpret, and/or use the equation, graph, or table of a linear function.
- Translate from one representation of a linear function to another (i.e., graph, table, and equation).

Sample Problems:

1. Tim’s scores for the first 5 times he played a video game are listed below.
   4,526  4,599  4,672  4,745  4,818
   Tim’s scores follow a pattern. Which expression can be used to determine his score after he played the video game \( n \) times?
   a. 73\( n \) + 4,453
   b. 73\( (n + 4,453) \)
   c. 4,453\( n \) + 73
   d. 4,526\( n \)

2. A pizza restaurant charges for pizzas and adds a delivery fee. The cost (c), in dollars, to have any number of pizzas (p) delivered to a home is described by the function \( c = 8p + 3 \). Which statement is true?
   a. The cost of 8 pizzas is $11
   b. The cost of 3 pizzas is $14
   c. Each pizza costs $8 and the delivery fee is $3
   d. Each pizza costs $3 and the delivery fee is $8.
3. The table below shows values of y as a function of x.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>14</td>
<td>55</td>
</tr>
<tr>
<td>26</td>
<td>100</td>
</tr>
<tr>
<td>34</td>
<td>130</td>
</tr>
</tbody>
</table>

Which linear equation best describes the relationship between x and y?

a.  \( y = 2.5x + 5 \)

b.  \( y = 3.75x + 2.5 \)

c.  \( y = 4x + 1 \)

d.  \( y = 5x \)

4. Which graph shows y as a function of x?
5.

The graph of a function is shown below.

Which value is not in the range of the function?

A. 0
B. 3
C. 4
D. 5

A1.2.2 Coordinate Geometry
• Identify, describe, and/or use constant rates of change.
• Apply the concept of linear rate of change (slope) to solve problems.
• Write or identify a linear equation when given
  • the graph of the line,
  • two points on the line, or
  • the slope and a point on the line.
Note: Linear equation may be in point-slope, standard, and/or slope-intercept form.
• Determine the slope and/or y-intercept represented by a linear equation or graph.
• Draw, identify, find, and/or write an equation for a line of best fit for a scatter plot.

Sample Problems:

6. Jeff’s restaurant sells hamburgers. The amount charged for a hamburger (h) is based on the cost for a plain hamburger plus an additional charge for each topping (t) as shown in the equation below.

   \[ H = 0.06t + 5 \]

What does the number 0.06 represent in the equation?

   a. the number of toppings  
   b. the cost of a plain hamburger  
   c. the additional cost of each topping  
   d. the cost of a hamburger with 1 topping

7. A juice machine dispenses the same amount of juice into a cup each time the machine is used. The equation below describes the relationship between the number of cups (x) into which juice is dispensed and the gallons of juice (y) remaining in the machine.

   \[ x + 12y = 180 \]

How many gallons of juice are in the machine when it is full?

   a. 12  
   b. 15  
   c. 168  
   d. 180

8. 
A ball rolls down a ramp with a slope of $\frac{2}{3}$. At one point the ball is 10 feet high, and at another point the ball is 4 feet high, as shown in the diagram below.

What is the horizontal distance \( x \), in feet, the ball traveled as it rolled down the ramp from 10 feet high to 4 feet high?

A. 6
B. 9
C. 14
D. 15
A graph of a linear equation is shown below.

Which equation describes the graph?

A. \( y = 0.5x - 1.5 \)
B. \( y = 0.5x + 3 \)
C. \( y = 2x - 1.5 \)
D. \( y = 2x + 3 \)
10.
The scatter plot below shows the cost \( y \) of ground shipping packages from Harrisburg, PA, to Minneapolis, MN, based on the package weight \( x \).

Which equation best describes the line of best fit?

A. \( y = 0.37x + 1.57 \)
B. \( y = 0.37x + 10.11 \)
C. \( y = 0.68x + 2.32 \)
D. \( y = 0.68x + 6.61 \)
A1.2.3 Data Analysis

- Calculate and/or interpret the range, quartiles, and interquartile range of data.
- Estimate or calculate to make predictions based on a circle, line, bar graph, measures of central tendency, or other representations.
- Analyze data, make predictions, and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, scatter plots, measures of central tendency, or other representations).
- Make predictions using the equations or graphs of best-fit lines of scatter plots.
- Find probabilities for compound events (e.g., find probability of red and blue, find probability of red or blue) and represent as a fraction, decimal, or percent.

Sample Problems:

11.

The daily high temperatures, in degrees Fahrenheit (°F), of a town are recorded for one year. The median high temperature is 62°F. The interquartile range of high temperatures is 32. Which is most likely to be true?

A. Approximately 25% of the days had a high temperature less than 30°F.
B. Approximately 25% of the days had a high temperature greater than 62°F.
C. Approximately 50% of the days had a high temperature greater than 62°F.
D. Approximately 75% of the days had a high temperature less than 94°F.
12. Vy asked 200 students to select their favorite sport and then recorded the results in the bar graph below.

![Bar Graph](image)

Vy will ask another 80 students to select their favorite sport. Based on the information in the bar graph, how many more students of the next 80 asked will select basketball rather than football as their favorite sport?

A. 10  
B. 20  
C. 25  
D. 30

13. The points scored by a football team are shown in the stem-and-leaf plot below.

![Stem-and-Leaf Plot](image)

What was the median number of points scored by the football team?

A. 24  
B. 27  
C. 28  
D. 32
14. John recorded the weight of his dog Spot at different ages as shown in the scatter plot below.

Based on the line of best fit, what will be Spot's weight after 18 months?

A. 27 pounds
B. 32 pounds
C. 36 pounds
D. 50 pounds

15. A number cube with sides labeled 1 through 6 is rolled two times, and the sum of the numbers that end face up is calculated. What is the probability that the sum of the numbers is 3?

A. \( \frac{1}{18} \)
B. \( \frac{1}{12} \)
C. \( \frac{1}{9} \)
D. \( \frac{1}{2} \)
**Constructed Response Questions**

The Algebra 1 Keystone Exam will have two different types of constructed response questions. Both types of constructed response questions will be scored on a scale ranging from 0-4 points.

**Scaffolding Completion Questions:** These questions will elicit two to four distinct responses from a student. No extraneous work or explanation will be scored.

Sample:

Keng creates a painting on a rectangular canvas with a width that is four inches longer than the height, as shown in the diagram below.

```
+---------+
|         |
|         |
|         |
|         |
+---------+
```

A. Write a polynomial expression, in simplified form, that represents the area of the canvas.

**Extended Scaffolding Completion Questions:** These questions require students to respond with extraneous work or explanation for a part of the question. For example, the student may be asked to “Show all work”, “Explain your work”, or “What is the error in the reasoning”.

Sample:

Keng is unhappy with his 3-inch-wide frame, so he decides to put a frame with a different width around his canvas. The total area of the canvas and the new frame is given by the polynomial $h^2 + 8h + 12$, where $h$ represents the height of the canvas.

C. Determine the width of the new frame. Show all your work. Explain why you did each step.