Simplifying Exponential Expressions
Exponential Notation

Base $^\text{Exponent}$

Base raised to an exponent

Example: What is the base and exponent of the following expression?

$7^2$

7 is the base

2 is the exponent
Goal

To write simplified statements that contain distinct bases, one whole number in the numerator and one in the denominator, and no negative exponents.

Ex:

\[
\left(\frac{9^{-1} a^4 b^{-3}}{6a^2 b^{-1} c^{-2}}\right)^2 = \frac{9b^8 c^4}{4a^{12}}
\]
Multiplying Terms

When we are multiplying terms, it is easiest to break the problem down into steps. First multiply the number parts of all the terms together. Then multiply the variable parts together.

Examples:

a. \((4x)(-5x) = (4 \cdot -5)(x \cdot x) = -20x^2\)

b. \((5z2)(3z)(4y) = (5 \cdot 2 \cdot 3 \cdot 4)(y \cdot z \cdot z) = 120yz^2\)

Only the z is squared
Evaluate the following *without a calculator*:

$$3^4 = 81 \div 3$$

$$3^3 = 27 \div 3$$

$$3^2 = 9 \div 3$$

$$3^1 = 3 \div 3$$

Describe a pattern and find the answer for:

$$3^0 = 1 \div 3$$
Zero Power

Anything to the zero power is one

\[ a^0 = 1 \]

Can “a” equal zero?

No.

You can’t divide by 0.
Exploration

Simplify:

\[ x^3 \cdot x^4 \]

Use the definition of exponents to expand

\[ XXX \cdot XXXXX \]

\[ x^{3+4} \]

Notice (from the initial expression) 3+4 is 7!

There are 7 “x” variables

\[ x^7 \]
Product of a Power

\[ a^m \cdot a^n = a^{m+n} \]

If you multiply powers having the same base, add the exponents.
Example

Simplify:

\[ x^2 \cdot 3 \cdot x^9 \cdot y^0 \]

Add the exponents since the bases are the same.

\[ x^{2+9} \cdot 3 \cdot 1 \]

Anything raised to the 0 power is 1.

\[ 3x^{11} \]
Practice

Simplify the following expressions:

1) \( x \cdot x^5 = x^6 \)

2) \( 2 \cdot x^3 \cdot z^0 \cdot 3 \cdot x^2 = 6x^5 \)

3) \( (-9x^3 y^5)(4x^2 y^4) = -36x^5 y^9 \)
Exploration

Simplify:

\[(x^3)^5\]

Adding 3 five times is equivalent to multiplying 3 by 5. The same exponents from the initial expression!

Use the definition of exponents to expand

The Product of a Power Rule says to add all the 3s
Power of a Power

$(a^m)^n = a^{m\cdot n}$

To find a power of a power, multiply the exponents.
Example

Simplify:

\[2s \left(s^2\right)^6 \left(t^3\right)^3 \cdot 4t^2\]

Multiply the powers of a exponent raised to another power

\[2s \cdot s^{2 \cdot 6} \cdot t^{3 \cdot 3} \cdot 4t^2\]

Any base without a power, is assumed to have an exponent of 1

\[2s^1 \cdot s^{12} \cdot t^9 \cdot 4t^2\]

Multiply numbers without exponents and add the exponents when the bases are the same

\[2 \cdot 4s^{1+12} \cdot t^{9+2}\]

\[8s^{13} \cdot t^{11}\]
Practice

Simplify the following expressions:

1) \( (y^2)^4 = y^8 \)

2) \( (a^4)^2 (a^3)^5 = a^{23} \)

3) \( x(y^5)^2 x^4 (x^2)^6 y = x^{17} y^{11} \)
Exploration

Simplify:

\[ \left( z^2 x \right)^5 \]

The Product of a Power Rule says to add the exponents with the same bases.

\[ z^2 x \cdot z^2 x \cdot z^2 x \cdot z^2 x \cdot z^2 x \]

Adding 2 five times is equivalent to multiplying 2 by 5.

Use the definition of exponents to expand.

Notice: Both the \( z^2 \) and \( x \) were raised to the 5\(^{th} \) power!
Power of a Product

\[(a \cdot b)^m = a^m \cdot b^m\]

If a base has a product, raise each factor to the power
Example

Simplify:

\[ (-3x)^2 \left(-2xy^4\right)^5 \]

\[ \left(-3\right)^2 \cdot x^2 \cdot \left(-2\right)^5 \cdot x^5 \cdot y^{4 \cdot 5} \]

\[ 9 \cdot x^2 \cdot (-32) \cdot x^5 \cdot y^{20} \]

\[ 9 \cdot (-32) \cdot x^{2+5} \cdot y^{20} \]

\[ -288x^7 y^{20} \]

Everything inside the parentheses is raised to the exponent outside the parentheses.

Multiply the powers of a base raised to another power.

Multiply numbers without exponents and add the exponents when the bases are the same.
Simplify the following expressions:

1) \((- pqr)^5 = - p^5 q^5 r^5\)

2) \((-2ab^2)^4 (-2a^3)^5 = -512a^{19}b^8\)

3) \(-2x (3x^2yz^4)^3 = -54x^7y^3z^{12}\)
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$125x^3$</td>
<td>9.</td>
</tr>
<tr>
<td>2.</td>
<td>$64d^6$</td>
<td>10.</td>
</tr>
<tr>
<td>3.</td>
<td>$a^7b^7c$</td>
<td>11.</td>
</tr>
<tr>
<td>4.</td>
<td>$64m^6n^6$</td>
<td>12.</td>
</tr>
<tr>
<td>5.</td>
<td>$100x^2y^2$</td>
<td>13.</td>
</tr>
<tr>
<td>6.</td>
<td>$-r^5s^5t^5$</td>
<td>14.</td>
</tr>
<tr>
<td>7.</td>
<td>$27b^4$</td>
<td>15.</td>
</tr>
<tr>
<td>8.</td>
<td>$-4x^7$</td>
<td></td>
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</tbody>
</table>
Complete the tables (with fractions) by finding the pattern.

<table>
<thead>
<tr>
<th>$5^n$</th>
<th>$5^{-n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^5$</td>
<td>3125</td>
</tr>
<tr>
<td>$5^4$</td>
<td>625</td>
</tr>
<tr>
<td>$5^3$</td>
<td>125</td>
</tr>
<tr>
<td>$5^2$</td>
<td>25</td>
</tr>
<tr>
<td>$5^1$</td>
<td>5</td>
</tr>
<tr>
<td>$5^0$</td>
<td>1</td>
</tr>
<tr>
<td>$5^{-1}$</td>
<td>1/5</td>
</tr>
<tr>
<td>$5^{-2}$</td>
<td>1/25</td>
</tr>
<tr>
<td>$5^{-3}$</td>
<td>1/125</td>
</tr>
<tr>
<td>$5^{-4}$</td>
<td>1/625</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\frac{1}{2^n}$</th>
<th>$\frac{1}{2^{-n}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2^5}$</td>
<td>$\frac{1}{32}$</td>
</tr>
<tr>
<td>$\frac{1}{2^4}$</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>$\frac{1}{2^3}$</td>
<td>$\frac{1}{8}$</td>
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<tr>
<td>$\frac{1}{2^2}$</td>
<td>$\frac{1}{4}$</td>
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<tr>
<td>$\frac{1}{2^1}$</td>
<td>$\frac{1}{2}$</td>
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<tr>
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<tr>
<td>$\frac{1}{2^{-1}}$</td>
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<td>8</td>
</tr>
<tr>
<td>$\frac{1}{2^{-4}}$</td>
<td>16</td>
</tr>
</tbody>
</table>
Negative Powers

A simplified expression has no negative exponents.

\[ a^{-m} = \frac{1}{a^m} \]

Negative Exponents “flip” and become positive.

1

\[ a^{-m} = \frac{1}{a^m} = a^m \]

A simplified expression has no negative exponents.
Example

Simplify:

\[4a^{-10} b^3 5a^4\]

\[4 \cdot 5a^{-10+4} b^3\]

\[20a^{-6} b^3\]

\[\frac{20b^3}{a^6}\]

All of the old rules still apply for negative exponents.

Flip ONLY the thing with the negative exponent to the bottom and the exponent becomes positive.

This is not simplified since there is a negative exponent.
Example

Simplify:

\[
\frac{12x^2y^{-1}}{8x^{-3}} \div \frac{12x^2x^3}{8y^1} = \frac{12x^{2+3}}{8y} \div 4
\]

\[
\frac{3x^5}{2y}
\]

Everything with a positive exponent stays where it is.

Since all of the negative exponents are gone, apply all of the old rules to simplify.

Everything with a negative exponent is flipped and exponent becomes positive.
Practice

Simplify the following expressions:

1) \( 8^{-3} = \frac{1}{512} \)

2) \( \frac{6x^2}{4x^{-5}y^3} = \frac{3x^7}{2y^3} \)

3) \( 3x^{-3}y^8x^{-4} = \frac{3y^8}{x^7} \)

4) \( (2a^{-3}b)^{-2} = \frac{a^6}{4b^2} \)
Exploration

Simplify:

\[ \frac{x^{10}}{x^6} \]

Since everything is multiplied, you can cancel common factors.

The 6 “x”s in the denominator cancel 6 out of the 10 “x”s in the numerator. This is the same as subtracting the exponents from the initial expression!

Only 4 “x”s remain in the numerator.

\[ x^{10-6} = x^4 \]
Quotient of a Power

To find a quotient of a power, subtract the denominator’s exponent from the numerator’s exponent if the bases are the same.

\[
\frac{a^m}{a^n} = a^{m-n}
\]

where \( a \neq 0 \).
Example

Simplify:

\[
\frac{2x^6y^1}{6x^2y^3}
\]

Divide the base numbers first

\[
\frac{2}{6} \cdot \frac{x^{6-2}}{y^{1-3}}
\]

Not simplified since there is a negative exponent

\[
\frac{1}{3}x^4y^{-2}
\]

Flip any negative exponents

\[
\frac{x^4}{3y^2}
\]

Subtract the exponents of the similar bases since there is division
Practice

Simplify the following expressions:

1) \( \frac{a^6 b^0}{5a^3} = \frac{a^3}{5} \)

2) \( \frac{12x^6}{4xy^{12}} = \frac{3x^5}{y^{12}} \)

3) \( \frac{14x^9 y^3}{4x^3 y} = \frac{7x^6 y^2}{2} \)
Exploration

Simplify:

Use the definition of exponents to expand

\[
\left( \frac{a}{b} \right)^6 \]

Multiply the fractions

Use the definition of exponents to rewrite.

Notice: Both the numerator and denominator were raised to the 6th power!
To find a power of a quotient, raise the denominator and numerator to the same power.

\[
\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}
\]

\(b \neq 0\)
Example

Simplify:

\[ \left( \frac{3}{y} \right)^{-2} \cdot \left( \frac{2x^2 y^7}{x^5} \right)^3 \]

\[ = \frac{3^{-2}}{y^{-2}} \cdot \frac{2^3 x^{2 \cdot 3} y^{7 \cdot 3}}{x^{5 \cdot 3}} \]

\[ = \frac{y^2}{3^2} \cdot \frac{8x^6 y^{21}}{x^{15}} \]

\[ = \frac{y^2 \cdot 8x^6 y^{21}}{3^2 \cdot x^{15}} \]

\[ = \frac{8y^{2+21}}{9x^{15-6}} \]

\[ = \frac{8y^{23}}{9x^9} \]

Everything in the fraction is raised to the power outside the parentheses.

Multiply the fractions

Subtract the exponents when there is division, and add when there is multiplication.
Practice

Simplify the following expressions:

1) \( \left( \frac{a^3}{bc^0} \right)^8 = \frac{a^{24}}{b^8} \)

2) \( \left( \frac{-2x}{y^2} \right)^{-4} = \frac{y^8}{16x^4} \)

3) \( \left( \frac{s^{-2}f^5}{zr^4} \right)^7 = \frac{f^{35}}{r^{28}s^{14}z^7} \)